Talk in honor of Albrecht Unsöld's 100th anniversary

Physics of stellar atmospheres – new aspects of old problems

Bodo Baschek

Zentrum für Astronomie der Universität Heidelberg
Institut für Theoretische Astrophysik
Albert-Ueberle-Strasse 2, D-69120 Heidelberg
`baschek@ita.uni-heidelberg.de`

Abstract

This contribution on the occasion of Albrecht Unsöld's (1905–1995) 100th anniversary comprises selected data and aspects of the biography and scientific achievements of this pioneer of stellar atmospheres physics, followed by a short outline of the development in this field over 75 years. Concerning present-day and future possibilities to obtain high-resolution spectra in the infrared, the theoretical aspects specific for this range to the physics of stellar atmospheres – such as stimulated emission, refractive and dispersive media, and metal optics for partially ionized low-conductivity gases – are briefly discussed.

1 Introduction

By this contribution on the occasion of his one hundredth birthday the Astronomical Society (Astronomische Gesellschaft) pays homage to Albrecht Unsöld (1905–1995), pioneer of the physics of stellar atmospheres and honorary member of the Society since 1989.

In this talk only selected aspects of A. Unsöld's vita, his scientific career and his scientific achievements and bibliography will be given; additional aspects and further biographic data may be found e.g. in the obituaries (Baschek 1996, Weidemann 1996, Seaton 1997) which were written about ten years ago.

The larger part of this contribution, however, will be devoted to discussions of some old and new problems in the field of stellar atmosphere physics. In his famous book "Physics of stellar atmospheres" (1938, 1955) A. Unsöld laid the foundations of the work in this field for many decades so that it seems an appropriate choice here

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to first give a brief outline of the development in this field over about 75 years and then turn to present-day aspects of the physics of stellar atmospheres on the basis of Unsöld's book.

While at that time the theory was mostly oriented by the interpretation of observations in the optical range, the recent achievements in observational techniques in "new" spectral ranges , in accordance with this meeting's motto "The many facets of the universe – *revelations by new instruments*", strongly influence the modelling of stellar atmospheres and quantitative abundance determinations. In particular the infrared, where e.g. by the Spitzer Space Telescope or by ALMA, high-resolution spectroscopic observations become available, demands a theoretical frame which is adequate to calculate *infrared*-dominated radiation on a level of sophistication and accuracy that presently is standard for the analysis of optical radiation. This "challenge of the infrared" has to be met by a critical re-examination of the classical assumptions made in Unsöld's "Physics of stellar atmospheres", by expanding them where necessary, and by working out new concepts so that we may experience a renaissance of Albrecht Unsöld's work on the physics of stellar atmospheres.

2 Albrecht Unsöld, a pioneer of stellar atmosphere physics

Born in Bolheim (Württemberg) on 20 April 1905, Albrecht Unsöld was only twenty-two years old when in 1927 he was awarded his doctorate (Dr. phil.) from the University of München (Munich); his thesis adviser was as Arnold Sommerfeld. In 1932, at the age of 27 years he became full professor (Ordinarius) of theoretical physics at the University of Kiel, from where he officially retired (Professor emeritus) in 1973. After his retirement he continued his scientific activity for many years. Unsöld died on 23 September 1995 in Kiel.

2.1 Selected publications

Unsöld's first publications 1925/27 dealt with quantum mechanical problems, in 1927 his first *"astrophysical"* publication, on the structure of the Fraunhofer lines and the dynamics of the solar chromosphere [1], appeared. It was followed by an impressive number of publications, mostly applying quantum physics to the structure of the atmospheres of the sun and the stars and to the analysis of their spectra, of which only very few of the important ones can be mentioned here.

Unsöld's paper of 1928 on the structure of the Fraunhofer lines and the quantitative spectral analysis of the solar atmosphere [2] comprises one of the first determinations of the *solar chemical composition*, and his publications 1931/32 on the convection in the solar atmosphere [3] identified the *hydrogen convection zone* as source of an import energy transport.

During the Second World War, in a series of papers Unsöld published 1941/44 the quantitative spectral analysis of the *B0 Star τ Scorpii* [5] based upon model atmosphere techniques. τ Sco is a star considerably hotter than the sun with a markedly different spectrum, in particular the lighter elements such as He, C, N, O which are not easily accessible in the sun, are represented by many lines in the B star.

Unsöld's book *"Physics of Stellar Atmospheres"* (1938) and its substantially enlarged second edition (1955) [4] served generations of researchers in this field as "Bible".

In 1967 the first edition of Unsöld's book *"The new cosmos"*, a concise and comprehensive introduction to the *entire* field of astronomy and astrophysics [6], appeared in German (Spanish and Japanese translations 1968, English translation 1969). This book, in updated versions, is still available as an introductory textbook to astronomy and astrophysics [6′].

In the two decades from about 1950 to 1970, in many PhD theses at the Kiel Institute under Unsöld's supervision the spectra of the *sun* and various types of *stars*, distributed over the Hertzsprung-Russell diagram, have been analyzed by sophisticated model atmosphere techniques (*"Feinanalyse"*) and their chemical compositions been determined.

2.2 Spectra and atomic data

Clearly, any progress of the theoretical physics of stellar atmospheres is on the one hand intimately interwoven with the development of observational techniques, in particular that of obtaining high-resolution stellar spectra, and on the other hand with the progress in the field of atomic physics and spectroscopy in order to cover the needs of the numerous atomic data required for the quantitative analysis of stellar spectra.

2.2.1 Observations: high-resolution spectra

For many decades, since the earliest phases of the development of the stellar atmosphere physics, the largest telescopes and powerful spectrographs were concentrated in the United States of America so that it was vital for the work of A. Unsöld to have close contacts to American scientists in order to have some access to high-resolution spectral observations.

Unsöld's first longer stay in the United States was realized 1928/29 as Fellow of the Rockefeller Foundation at the Mount Wilson Observatory. In 1939 he spent some months before the outbreak of the Second World War as guest professor at Chicago with visits at the Yerkes and the McDonald Observatory. Here he worked together with Otto Struve, professor of astrophysics at the University of Chicago, director of the Yerkes Observatory, and since 1939 also director of the newly inaugurated McDonald Observatory of the University of Texas at Austin. Struve and Unsöld decided to obtain high-resolution spectra of the sharp-lined B star τ Sco with the new Cassegrain and Coude spectrographs of the 82-inch telescope over a spectral range as wide as possible at that time (3320–6560Å). These spectra, together with observations previously taken by O. Struve and his collaborators, were the basis of the detailed analysis τ Sco by Unsöld, carried out during the war [5].

Already five years after the end of the war Unsöld began to establish again contacts to American scientists on the occasion of a research stay as guest professor at

Haverford (Pennnsylvania). Longer visits to Pasadena and the Lick Observatory in 1957, and a guest professorship at the California Institute of Technology, Pasadena, in 1961 followed.

In this context it should be mentioned that Jesse L. Greenstein's famous "Abundance project" (1957–1970) gave many post-docs from all over the world, including students of Unsöld's, the opportunity to spend some time at Pasadena in order to discuss the problems of the chemical compositions of the stars and their evolution and also to observe at the Mount Wilson Observatory and to obtain high-dispersion spectra.

2.2.2 Atomic data

The almost insatiable need for atomic data by the astrophysicists for their quantitative analyses of stellar spectra such as energy levels and line identifications, oscillator strengths or f-values, continuous absorption cross-sections, lifetimes and damping constants, required a word-wide and at least partially organized collaboration with experimental and theoretical atomic physicists.

Regarding Unsöld's institute at Kiel, a particularly fruitful close collaboration and scientific interaction took place over about three decades with Institute for Experimental Physics under its director W. Lochte-Holtgreven, located next-door on the university campus.

As an example for this successful cooperation the significant contribution to the solution of the long-standing *"iron-problem"* should be mentioned: the determination of the solar iron abundance from the photospheric spectrum, from lines in the extreme ultraviolet, and from meteorites yielded results with discrepancies of up to an order of magnitude. Experimentally determined oscillator strengths of neutral and singly ionized Fe-lines in combination with calculations of these lines by theoretical models of the solar photosphere resolved these differences (for more details see Unsöld's article of 1971 [7]).

2.3 Computers for stellar atmosphere physics at Kiel

Since 1958 electronic computers for stellar atmosphere calculations have been available at Kiel, the first one being a Zuse Z 22, followed by an Electrologica X 1. The use of computers, beginning world-wide at astrophysical institutions about the same time, had an enormous impact on the development of the field of stellar atmospheres, as it did on almost any branch of science.

First of all, the computations could be carried out considerably faster, e.g. the typical time required for the calculation of one model atmosphere "by hand" of about *1 month* was reduced to the order of *1 hour* by the Z 22. Furthermore, many approximations, dictated only by the necessity of saving computational time, became superfluous, and large parts of the spectral analyses and abundance determinations could be carried out more or less "automatically", and last but not least, new and more complex problems became accessible to modelling.

A. Unsöld himself did not personally participate actively in the code development, however, he supported and encouraged the work at his institute at Kiel with great interest and some scepticism.

Beginning with the First Harvard-Smithsonian conference on stellar atmospheres in 1964, frequent international meetings of the groups engaged in code development for the calculation of model stellar atmospheres and of spectral lines, for the determination of element abundances, and for the associated physics input (ionization equilibria, partition functions etc.) took place on which methods and results could be compared and critically be discussed.

A detailed review of the achievements over *fifty years* in the field of stellar atmospheres and determination of element abundances was presented by Albrecht Unsöld himself in 1978 in his introductory talk [8] at the International Colloquium on "The elements and their isotopes in the universe" at Liège.

Selected bibliography of A. Unsöld

[1] Unsöld, A., 1927, Über die Struktur der Fraunhoferschen Linien und die Dynamik der Sonnenchromosphäre, Z. Physik 44, 793

[2] Unsöld, A., 1928, Über die Struktur der Fraunhoferschen Linien und die quantitative Spektralanalyse der Sonnenatmosphäre, Z. Physik 46, 765

[3] Unsöld, A., 1930, Konvektion in der Sonnenatmosphäre, Z. Astrophys. 1, 1; 1931, Konvektion in der Sonnenatmosphäre II, Z. Astrophys. 2, 209

[4] Unsöld, A., 1938, Physik der Sternatmosphären (mit besonderer Berücksichtigung der Sonne), 1st ed.; 1955, 2nd ed.; 1968, corr. 2nd printing, Springer, Berlin

[5] Unsöld, A., 1941, Quantitative Spektralanalyse des B0-Sternes τ Scorpii I, Z. Astrophys. 21, 1; II, Z. Astrophys. 21, 22; 1942, III, Z. Astrophys. 21, 229; 1944, IV, Z. Astrophys. 23, 75

[6] Unsöld, A., 1967, Der neue Kosmos, 1st ed., Heidelberger Taschenbücher 16/17, Springer, Berlin; 1969, The new cosmos, 1st ed., Springer, New York

[6$'$] Unsöld, A., Baschek, B., 2005, Der neue Kosmos – Einführung in die Astronomie und Astrophysik, 7th ed. (corr. 2nd printing), Springer, Berlin; 2005, The new cosmos – an introduction to astronomy and astrophysics, 5th ed. (corr. 2nd printing), Springer, Berlin

[7] Unsöld, A., 1971, Abundance of iron in the photosphere, Phil. Trans. Roy. Soc. Lond. A. 270, 23

[8] Unsöld, A., 1979, Introduction: A Fifty Years Retrospect, 22nd Liège Internat. Astrophys. Coll. "Les elements et leurs isotopes dans l'univers", p. 7

3 Seventy-five years stellar atmospheres

To illustrate the development of the physics of stellar atmospheres, but also to show the continuity of the work by A. Unsöld and his students over about 75 years, we pick out the problems of the calculation of spectral lines and of the inclusion of the very large number of spectral lines which is demanded by an accurate calculation of a model stellar atmosphere. Subsequently a brief overview over the development of the modelling of stellar atmospheres is given.

3.1 On spectral lines

For more than three decades, before electronic computers became widely available, the calculation even of a single line profile, i.e. of the frequency dependence or shape of a spectral line, was a tedious procedure requiring substantial numerical effort. Therefore abundance analyses were mostly based on using *equivalent widths*, i.e. frequency-*integrated* line profiles, and on *curves of growth*.

At present the inclusion of more than 10^6 spectral lines as part of the absorption coefficient in the modelling stellar atmospheres is more or less standard, at least for static atmospheres. The *direct* calculation of such a *synthetic spectrum* of many largely overlapping lines became possible, of course, only by the use of computers (cf. also Sect. 2.3).

It is impressive to realize the enormous increase in computer speed from the early 1960s until today: for example, the computation of the Mg II blend at $\lambda\,4481\,\text{Å}$ in an F-type star which consists of only *three* components, two Mg II lines and one Ti I line, took 4 minutes (!) of computer time on the Electrologica X 1 at Kiel, cf. Baschek & Traving (1962, Fig. 2). By the way, this may be regarded as one of the first calculations of a synthetic spectrum, albeit only of a very small wavelength piece.

An alternative to the direct inclusion of very many lines is the concept of *opacity distribution* functions (ODF) in which, for any given wavelength λ, the individual "deterministic" absorption by the lines is replaced by a smooth distribution of the line strengths in a sufficiently small wavelength interval. A forerunner of this concept is the *"fat line"*, introduced by Labs (1951) at Kiel, where all lines in a small interval are added together to form a *single* line with the same absorption.

Wehrse et al. (1998) showed that the assumption that the spectral lines follow a *Poisson point process* is a good and flexible statistical approximation, allowing many operations to be carried out analytically. A generalization of the opacity distribution functions to (differentially) *moving* media was given by Baschek et al. (2001) by introducing as second *variable* – in addition to λ – the *width* of a wavelength interval over which the line absorption is averaged.

3.2 On the modelling of stellar atmospheres

In the *beginning* the analysis of stellar spectra by means of model atmospheres was based on high-resolution spectra in the *optical*, ranging from the near ultraviolet to the visual, and later extending to the red and near infrared. The model atmospheres had to be approximated by plane-parallel (i.e. thin, compact, 1D), homogeneous and static stellar atmospheres in local thermodynamic equilibrium (LTE), sometimes also including scattering.

At *present*, at least in principle, spectra can be obtained for the *entire* electromagnetic spectrum, and models can be constructed for extended (3D or 2D), inhomogeneous media in statistical equilibrium, described by rate equations ("non-LTE"). The medium may be differentially moving, turbulent and convective motions can be included, and relativistic velocities may occur; also time-dependent phenomena can be treated.

The objects are not restricted to stellar atmospheres (photospheres, chromospheres, coronas, stellar winds); accretion disks and nebulas are as well included.

There seems, however, hardly any astrophysical application which could not be modelled provided sufficiently powerful computers are available, and the impression is gained that all remaining problems can be solved by still more powerful computers. Nevertheless, the realization of a code for the "general problem" comprising all the cases listed seems still to take many years.

The recent rapid development of observational techniques, however, enforces a more and more accurate modelling e.g. of stellar atmospheres so that the question may be allowed, in how far astrophysicists are prepared to interpret the forthcoming observations in "new" spectral ranges such as e.g. the *infrared* by reasonably accurate models.

One possible approach to the infrared would be starting from the radio frequency range where stimulated emission governs the radiation field, and where refractive media, partially ionized gases of low conductivity are typical, but where a genuine radiative transport, i.e. for optically thick media, is not needed in practice.

Here the alternative approach is chosen, going from the optical range to longer wavelengths, i.e. starting from the "classical" stellar atmospheres (photospheres) where the transfer equation with absorption and re-emission is essential and optical radiation dominates. This means that e.g. in Unsöld's "Physics of stellar atmospheres" *all* classical assumptions have to be *critically re-examined* for their validity also in the infrared spectral range.

In the following section, we will discuss in how far the infrared range may be a challenge to present-day theoretical astrophysics. This discussion is based upon work in preparation by Wehrse & Baschek (2005).

4 The infrared spectral range – a challenge to theoretical astrophysics?

Recently the Spitzer Space Telescope (SST) began operation in the wavelength range $\lambda\lambda\,3$–180 µm, and soon ALMA, the Atacama Large Millimeter Array, will make the range $\lambda\lambda\,350$ µm–10 mm accessible so that high resolution spectra in the *infrared* and *microwave* region begin to become available.

Among the stellar atmospheres, objects with intense radiation in the infrared, suitable for high-resolution spectroscopy in the infrared and for accurate modelling are e.g.
(a) *M dwarf* atmospheres, some of which have chromospheres,
with temperatures $T \simeq (3\ldots4)\cdot10^3\,\mathrm{K}$ and electron densities
$N_\mathrm{e} \simeq 10^{13}\ldots10^{16}\,\mathrm{cm}^{-3}$,
and
(b) *white dwarf* atmospheres with $T \simeq 10^4\,\mathrm{K}$ and $N_\mathrm{e} \simeq 10^{14}\ldots10^{16}\,\mathrm{cm}^{-3}$.

For the purpose of comparisons and numerical estimates the optical, infrared, and radio ranges may be characterized by typical wavelengths λ_opt, λ_ir, and λ_radio, respectively:

$$\begin{array}{ccccc} \text{OPTICAL} & \Longrightarrow & \textbf{INFRARED} & \Longleftarrow & \text{RADIO} \\ \lambda_{\mathrm{opt}} \simeq & & \lambda_{\mathrm{ir}} \simeq & & \lambda_{\mathrm{radio}} \simeq \\ 0.1\,\mu\mathrm{m} & & 100\,\mu\mathrm{m} & & 10\,\mathrm{cm} \\ & \dfrac{\lambda_{\mathrm{ir}}}{\lambda_{\mathrm{opt}}} \simeq 10^{+3} & & \dfrac{\lambda_{\mathrm{ir}}}{\lambda_{\mathrm{radio}}} \simeq 10^{-3}\,. & \end{array}$$

In order to gain first insight into the characteristic properties of the infrared spectral range, we first introduce the parameter

$$\alpha = \frac{h\nu}{kT} = \frac{c_2}{\lambda T} \tag{1}$$

which essentially measures the ratio of the energy of a photon of frequency ν (or wavelength λ) to the mean thermal kinetic energy. Here h is the Planck constant, k the Boltzmann constant, and $c_2 = hc/k \simeq 1.44\,\mathrm{cm}\cdot\mathrm{K}$ Planck's radiation constant. Then – for *thermal* radiation – the ratio η of spontaneous to stimulated photons is

$$\eta = \frac{1}{\mathrm{e}^{\alpha} - 1}\,, \tag{2}$$

and the "classical" factor for stimulated emission, cf. Eqs.(8, 9), is

$$q_{\kappa,c} = 1 - \mathrm{e}^{-\alpha}\,. \tag{3}$$

For a typical temperature of $T = 4000\,\mathrm{K}$:

		optical	infrared	radio
α	$\simeq$	36	0.04	$4 \cdot 10^{-5}$
η	$\simeq$	$2 \cdot 10^{-16}$	27	$3 \cdot 10^{+4}$
$q_{\kappa,c}$	$\simeq$	1	0.04	$4 \cdot 10^{-5}$.

Noting that α drops from values $\gg 1$ in the optical range to values $\ll 1$ in the infrared, we see that the stimulated emission, characterized by η or $q_{\kappa,c}$, is unimportant in the ultraviolet and optical, but begins to exceed the spontaneous emission in the near infrared and dominates more and more towards longer wavelengths.

In the following we will give a short discussion on the rate equation and the radiative transfer equation for a two-level atom, on the problems for refractive and dispersive media, on the effects of the particle density on the radiation field, and on the general theoretical frame to adequately describe the interaction between infrared radiation and matter.

4.1 Two-level atom

In order to elucidate the essential terms characteristic for the strength of an infrared spectral line, it suffices to consider a simple model level, consisting of only two bound levels, a lower one "l" and an upper one "u", either of statistical weight 1. Their energy difference is $h\nu$, and their occupation numbers are N_l and N_u, respectively.

The *rate equation* for the stationary case is

$$N_\mathrm{l}\,(B_\uparrow J_\nu + C_\uparrow) = N_\mathrm{u}\,(A_\downarrow + B_\downarrow J_\nu + C_\downarrow) \tag{4}$$

where upward transitions (l→ u) are denoted by ↑ and correspondingly downward transitions by ↓. A and B are the Einstein coefficients obeying the relations

$$A_\downarrow = \frac{2h\nu}{c^2}\,B_\downarrow\,, \qquad B_\downarrow = B_\uparrow\,, \tag{5}$$

and J_ν is the mean intensity. For particles obeying a Maxwell-Boltzmann velocity distribution, collisional excitation and de-excitation rates are related by

$$C_\downarrow = C_\uparrow\,\mathrm{e}^\alpha\,. \tag{6}$$

When effects of refraction (cf. Sect. 4.2) are neglected, the *radiative transfer* equation

$$\frac{dI_\nu}{d\ell} = -\kappa_\nu I_\nu + \eta_\nu\,, \tag{7}$$

describing the change dI_ν of the monochromatic specific intensity I_ν over the element $d\ell$ of the light path (κ_ν absorption coefficient, η_ν emission coefficient), reads for the two-level atom

$$\begin{aligned}\frac{dI_\nu}{d\ell} &= -B_\uparrow\,(N_\mathrm{l} - N_\mathrm{u})\,I_\nu + A_\downarrow N_\mathrm{u}\\ &= -q_\kappa \cdot B_\uparrow N_\mathrm{l}\,I_\nu + A_\downarrow N_\mathrm{u}\,.\end{aligned} \tag{8}$$

In this formulation we consider the stimulated emission as negative absorption, characterized by the factor q_κ. The *solution* of Eqs. (4–6) can be written as

$$q_\kappa = \frac{N_\mathrm{l} - N_\mathrm{u}}{N_\mathrm{l}} = \frac{1 + \frac{C_\uparrow}{A_\downarrow}\,(\mathrm{e}^\alpha - 1)}{1 + \dfrac{\iota_\nu}{\mathrm{e}^\alpha - 1} + \frac{C_\uparrow}{A_\downarrow}\,\mathrm{e}^\alpha}\,, \tag{9}$$

where $\iota_\nu = J_\nu / B_\nu(T)$ is the mean intensity expressed in units of the Planck function $B_\nu(T)$.

If the radiation field is Planckian ($\iota_\nu = 1$) or if the collision rates dominate the radiation rates ($C_\uparrow/A_\downarrow \gg 1$) we recover from Eq. (9) the well-known "classical" LTE case, Eq. (3), which for the infrared, i.e. $\alpha \ll 1$, results in $q_{\kappa,c} \to \alpha$.

For arbitrary $C_\uparrow/A_\downarrow$, Eq. (9) reduces to $q_\kappa \to 1$ for $\alpha \gg 1$, i.e. stimulated emission becomes unimportant in the optical and ultraviolet. In the infrared and radio range ($\alpha \ll 1$), however,

$$q_\kappa \simeq \frac{1 + \alpha C_\uparrow/A_\downarrow}{1 + \iota_\nu/\alpha + C_\uparrow/A_\downarrow}\,. \tag{10}$$

Here the ratio $C_\uparrow/A_\downarrow$ explicitly enters the factor for stimulated emission. If furthermore neither $C_\uparrow/A_\downarrow$ is too large, nor ι_ν too small, Eq. (10) reduces to

$$q_\kappa \simeq \frac{\alpha}{\iota_\nu}\,. \tag{11}$$

This means e.g. for a partially ionized gas that in the *infrared*, as compared to the optical, the collisions are less important, deviations from the Planck field have a stronger influence, and the gas is more transparent.

4.2 Refractive and dispersive media

If the frequency ν approaches the plasma frequency ν_P or falls within a strong spectral line ("resonance"), the refractive index may substantially differ from its vacuum value 1. The plasma frequency is given by

$$\nu_\mathrm{P}^2 = \frac{1}{\pi}\frac{N_\mathrm{e}e^2}{m_\mathrm{e}} \tag{12}$$

where N_e is again the electron density, m_e the mass, and e the charge of the electron .

Considering the range of electron densities between $N_\mathrm{e} = 10^{11}$ and $10^{17}\,\mathrm{cm}^{-3}$, which comprises e.g. the range of M dwarf atmospheres, we find from Eq. (12) plasma frequencies corresponding to wavelengths between $\lambda_\mathrm{P} \simeq \lambda_\mathrm{radio} \simeq 10\,\mathrm{cm}$ and $\lambda_\mathrm{P} \simeq \lambda_\mathrm{ir} \simeq 100\,\mu\mathrm{m}$, i.e. plasma frequencies extending into the infrared.

If $n \neq 1$ (*refractive* medium), we may have to consider spatial variations along the – in general curved – light path. If furthermore n depends on the frequency ν (*dispersive* medium), so that $n = n(\ell,\nu)$, the equation of radiative transfer has to be modified by additional $\partial n/\partial \ell$ – terms, and additional equations for the light paths for each frequency ν (or the explicit eikonal equations) are required.

In the astrophysical literature, e.g. for application to radio waves in the solar corona, the equation of *radiative transfer* usually is taken in the version

$$n^2 \frac{d}{d\ell}\left(\frac{I_\nu}{n^2}\right) = -\kappa_\nu I_\nu + \eta_\nu \tag{13}$$

or, equivalently,

$$\begin{aligned} \frac{dI_\nu}{d\ell} &= -\kappa_\nu I_\nu + \eta_\nu + \frac{\partial \ln(n^2)}{\partial \ell}\, I_\nu \\ &= -\left(\kappa_\nu - \frac{2}{n}\frac{\partial n}{\partial \ell}\right) I_\nu + \eta_\nu\,, \end{aligned} \tag{14}$$

cf. Woolley & Stibbs (1953), Unsöld (1955, [4]), Oster (1963), Harris (1965), Zheleznyakov (1996), cf. also Cox & Giuli (1968), Weiss et al. (2004). According to Woolley & Stibbs (1953) the emissivity for local thermodynamical equilibrium is

$$\eta_\nu = n^2\,\kappa_\nu B_\nu(T) \tag{15}$$

with $B_\nu(T)$ again being the Planck function (Kirchhoff's law).

For a spatially varying refraction index a different treatment has recently been given e.g. by Khan & Thomas (2005), describing the photon transport in biological tissues for application to optical tomography in medicine.

The *phase* velocity of light in a refractive medium with an index of refraction n is

$$v_\phi = \nu\lambda = \frac{c}{n} = \frac{c}{\sqrt{\epsilon\mu}} \simeq \frac{c}{\sqrt{\epsilon}}\,, \tag{16}$$

where ϵ is the dielectric constant and $\mu \simeq 1$ the magnetic permeability, and the wavelength λ in the medium is related to the vacuum wavelength λ_0 by $\lambda = \lambda_0/n$ (note that $\nu = \nu_0$).

In a dispersive medium, particular for time-dependent problems, care has to be taken to replace the phase velocity by the *group* velocity $v_{\rm gr} = v_\phi - \lambda\, dv_\phi/d\lambda$ at the appropriate places.

4.3 Radiation field and particle density

An important parameter for describing the interaction of the radiation field, considered at a wavelength λ, with the atoms of type j of the gas is the number $\mathcal{N}_{\lambda,j}$ of atoms j in a volume $V_{\lambda,j} = a^3$ with $a \ll \lambda$. All atoms in this volume then "see" essentially e.g. the *same* electric field, and any *continuum* theory such as e.g. the classical electrodynamics, requires that

$$\mathcal{N}_{\lambda,j} \gg 1\,. \tag{17}$$

For numerical estimates, we somewhat arbitrarily choose $a = \lambda/10$ or $V_{\lambda,j} = (\lambda/10)^3$. As an example we again consider the *atmosphere* of an *M dwarf.* Here the *total* particle density is of the order of $N_{\rm t} = 10^{18}\,{\rm cm}^{-3}$ so that in the

infrared $\mathcal{N}_{\rm ir,t} \simeq 10^9$

whereas in the

optical $\mathcal{N}_{\rm opt,t} \simeq 1$.

Hence a gas with a total particle density $N_{\rm t} = 10^{18}\,{\rm cm}^{-3}$ can on the one hand be considered as a *diluted* gas for *optical* radiation and for its kinetic properties, and on the other hand has to be treated as a *dense* gas with respect to the *infrared* radiation.

At this place, we mention an interesting quantum optical phenomenon which is possible if $\mathcal{N}_{\lambda,k} \gg 1$, the *superradiance* where the spontaneous emission of all particles of the same state k occurs then *coherently*, and – depending on the occupation of the initial excitation states – may even be proportional to N_k^2 (cf. Dicke 1954, Brandes 2005). Since, however, this photon coherence takes place in a pulse of very short duration only, there seems to be no astrophysical application of this effect in sight.

4.4 Metal optics

From the discussion in Sects.4.1 to 4.3 it seems obvious that basically the theoretical frame for an adequate treatment of the interaction of the partially ionized gas of a stellar atmosphere with an infrared radiation field is the continuum optics of metals, albeit with *low* electric conductivity, based upon Maxwell equations and some material equations, combined with the quantum theory for the electrons and the polarizability of atoms.

In the *continuum* theory of metal optics (cf. Born & Wolf 1999) the matter is characterized by three material "constants" which, however, are functions of the frequency ν: the dielectric constant $\epsilon(\nu)$, the electric conductivity $\sigma(\nu)$, and the magnetic permeability $\mu(\nu)$. The conductivity and the dielectric constant enter by the characteristic combination $(\epsilon + i\,2\sigma/\nu) = \epsilon \cdot (1 + i\,\zeta)$, and the "astrophysical constraint" of low conductivity

$$\zeta = \frac{2\sigma}{\nu\epsilon} \ll 1 \tag{18}$$

would avoid that a strong "skin effect" occurs in the partially ionized gas. For astrophysical applications to stellar atmospheres we may furthermore assume the vacuum value $\mu = 1$ of the magnetic permeability.

The material constants can be expressed by the *optical "constants"*, i.e. the refractive index $n(\nu)$ and the absorption or attenuation index $k(\nu)$, and vice versa.

As far as the quantum theoretical part of the metal optics is concerned, there are no problems as long as the gas is sufficiently dilute so that essentially the quantum physics of more or less isolated, individual atoms can be applied. However, a consistent quantum theoretical treatment of the *collective* interaction of the atoms with the radiation field for application in astrophysics still seems difficult to work out.

We note that at present there is active discussion of *infrared* radiation problems going on in *physics*, e.g. of coherent and collective quantum optical effects (cf. Brandes 2005) and of optical tomography (cf. Khan & Thomas 2005).

An interesting example illustrating the problems of the interaction between atoms and light in the presence of collective effects is the long-standing discussion over almost one hundred years about how the *momentum* of a *photon* changes from its value $h/\lambda_0 = h\nu/c$ in the vacuum when it travels through a dispersive, dilute gas with index of refraction n. The debate was ended only recently by the experiment of Campbell et al. (2005) in which the change in the momentum of a *single* photon was measured directly: due to the collective reaction of all atoms the momentum of the photon in a dispersive medium is $n{\cdot}h/\lambda_0 = h/\lambda = n{\cdot}h\nu/c = h\nu/v_\phi$.

5 Outlook

It is surprising that the *infrared*, the "new" spectral range for high-resolution spectroscopy, requires a theoretical frame for adequately accurate analysis of the radiation which on the one hand can be described by the *"classical roots"* of the 19th century, e.g. by the fundamental work of R. Clausius & O.F. Mossotti (1850), J.C. Maxwell (1873), H.A. Lorentz & L. Lorenz (1880/81), H. Hertz (1888), and P. Drude

(1900), and which on the other hand involves not only the interaction of single atoms with the radiation field, but to some extent also the collective action of the atoms.

To the quantum physics part of the metal optics it was in particular Arnold Sommerfeld (1868–1951) who in the first half of the 20th century made substantial contributions, and we may be sure that his student Albrecht Unsöld was well familiar with this field. Nevertheless, in Unsöld's "Physics of stellar atmospheres" hardly any discussion of metal optics is found since the interpretation of ultraviolet and optical spectra requires a theory of the interaction of radiation with the diluted matter of stellar atmospheres in which the effects of metal optics are negligible.

The challenge of the infrared spectral range for the stellar atmosphere physics might, however, be met by expanding the "classical" assumptions made in Unsöld's "Physics of stellar atmospheres" so that also the infrared radiation could be as accurately analyzed as the optical, giving us thus the fascinating prospect of a renaissance of Unsöld's work on the physics of stellar atmospheres.

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References

Baschek, B., 1996, Mitt. Astron. Ges. 79, 11; Phys. Bl. Nr.9, 890

Baschek, B., Traving, G., 1962, Z. Astrophys. 54, 7

Baschek, B., Waldenfels, W. von, Wehrse, R., 2001, A&A 371, 1084

Born, M., Wolf, E., 1999, 7th ed., Principles of optics, Pergamon Press, Oxford

Brandes, T., 2005, Physics Reports 408, 315

Campbell, G.K., Leanhardt, A.E., Mun, J., Boyd, M., Streed, E.W., Ketterle, W., Pritchard, D.E., 2005, Phys. Rev. Letters 94, 170403

Cox, J.P., Giuli, R.T., 1968, Principles of stellar structure, Gordon and Breach

Dicke, R.H., 1954, Phys. Rev. 93, 99

Harris, E.G., 1965, Phys. Rev. 138 B, 479

Khan, T., Thomas, A., 2005, Optics Comm. 255, Issue 1–3, 130

Labs, D. 1951, Z. Astrophys. 29, 199

Oster, L., 1963, ApJ 138, 761; criticism: Cronyn, W.M., 1966, ApJ 144, 834; reply: Oster, L., 1966, ApJ 144, 838

Seaton, M., 1997, Astron. Geophys. 38, 37

Wehrse, R., Baschek, B., 2005, in preparation

Wehrse, R., Waldenfels, W. von, Baschek, B., 1998, J. Quant. Spectr. Rad. Transfer 60, 963

Weidemann, V., 1996, SuW 35, No. 3, 182; PASP 108, 553

Weiss, A., Hillebrandt, W., Thomas. H.-C., Ritter, H., 2004, Cox & Guili's principles of stellar structure, Cambridge Sci. Publishers

Woolley, R v.d.R., Stibbs, D.W.N., 1953, The outer layers of a star, Clarendon Press, Oxford

Zheleznyakov, V.V., 1996, Radiation in astrophysical plasmas, Kluwer Academic Publ., Dordrecht